Praxis Core Academic Skills for Educators

Math Review

Number and Quantity

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Numbers and Quantity

TOPICS

Integers

- *Basic Operations*
- *Factors*
- *Divisibility Rules*
- *Multiples*
- *Greatest Common Factor*
- *Least Common Multiple*
- *Exponents*
- *Square Roots*

Number Line

- *Numeration*
- *Place Value*
- *Ordering*
- *Sequences*

Order of Operations

Fractions

- *Simplifying*
- *Mixed Numbers*
- *Basic Operations*
- *Converting*

Decimals and Percents

- *Converting*
- *Basic Operations*

Reasoning

- *Metric System*
- *English System*
- *Converting Units of Measure*
- *Word Problems*

Numbers and Quantity

Properties of Integers

***Integers*:** **… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …** *0 is neither positive nor negative!*

negative positive

***Even Numbers*:** **… , -4, -2, 0, 2, 4, …** *0 is an even number!*

***Odd Numbers*:** **… , -5, -3, -1, 1, 3, 5, …**

***Addition of Integers*:**

even + even = even	***4 + 8 = 12***
odd + odd = even	***5 + 3 = 8***
odd + even = odd	***3 + 4 = 7***

***Multiplication of Integers*:**

even x even = even	***4 x 8 = 32***
odd x odd = odd	***5 x 3 = 15***
odd x even = even	***3 x 4 = 12***

***Consecutive Integers*:**

5, 6, 7, 8
1003, 1004, 1005, 1006, 1007
-12, -11, -10
n, n+1, n+2, n+3, … where n is any integer

***Problem 1*: If three consecutive integers sum to 150, what is the smallest of the three?**

***Problem 2*: If four consecutive odd integers sum to 64 what is the largest of the four?**

Numbers and Quantity

Arithmetic Word Problems

Approach:

1. ***Identify the "Given"***
2. ***Identify what is being "looked for"***
3. ***Using the "Given", draw conclusions***

Example Problem 1:

Mr. Jones is making cookies for his class. He has 20 students and plans on giving each student 3 cookies. However, on cookie day, some students were absent so that Mr. Jones could give each student exactly 4 cookies, with none left over. How many students were absent?

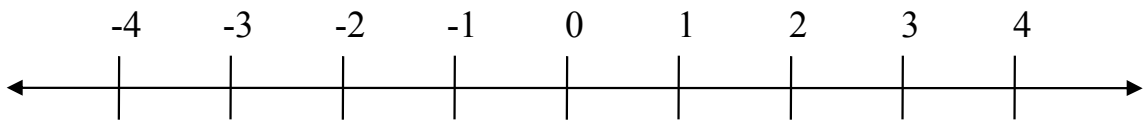
1. ***Identify the "Given"***
   20 students and he made 3 cookies per student
   Each student that was present received 4 cookies (none left over)
   Some students were absent on cookie day
2. ***Identify what is being "looked for"***
   Number students absent
3. ***Using the "Given", draw conclusions***
   Mr. Jones made 60 cookies total (20 students x 3 cookies each)
   Each student received 4 cookies, with none left over, so 60 / 4 = 15 students

***Problem 1*: Jim buys three erasers at the store, all the same price. He pays with two dollars. He receives two coins in change, a quarter and a dime. What was the price of a single eraser?**

***Problem 2*: Mr. Smith is selling apples. Each bag of apples costs $3.50. There are 10 apples in each bag. He also sells caramel treats for 45 cents each. One customer bought two bags of apples and some caramel treats for $8.35. How many caramel treats did the customer buy?**

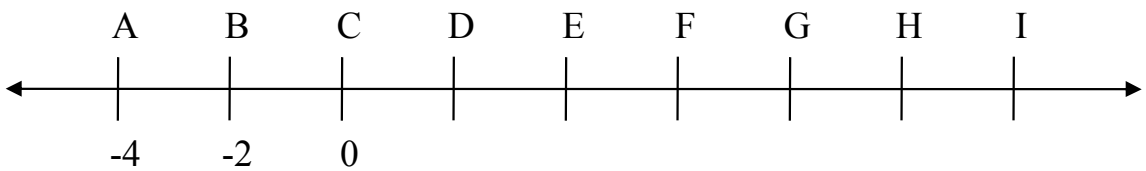
Numbers and Quantity

Number Lines



As you move to the left ←
Numbers DECREASE

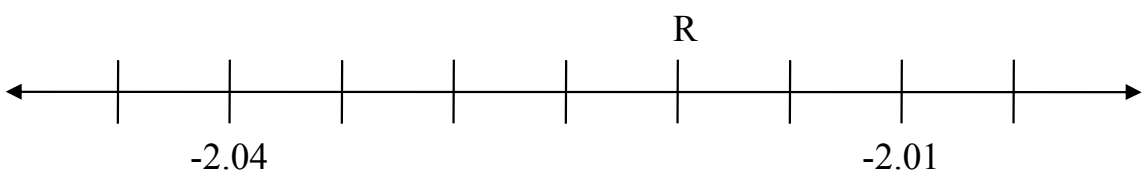
As you move to the right →
Numbers INCREASE



Problem 1: On the above number line, what is the value of G?

Problem 2: On the above number line, the ratio of AC to AE is equal to the ratio of CD to what?

NOTE: The tick marks do not have to be integer distances!



Problem 3: On the above number line, what is the value of R?

Numbers and Quantity

Squares and Square Roots

Know your Perfect Squares!

1,4,9,16,25,36,49,64,81,100,121,144

if $\sqrt{x} = y$, then $y^2 = x$

$\sqrt{49} = 7$, so $7^2 = 49$

$\sqrt{81} = -9$, so $(-9)^2 = 81$

Given a real number **n**

if $n > 1$, then $n^2 > n$ — *Example: Let n = 5, then 25 > 5*

if $0 < n < 1$, then $n^2 < n$ — *Example: Let n = 0.5, then 0.25 < 0.5*

$$\left(\frac{n}{d}\right)^2 = \frac{n^2}{d^2} \qquad \left(\frac{4}{9}\right)^2 = \frac{4^2}{9^2} = \frac{16}{81}$$

Numbers and Quantity

Fractions and Rational Numbers

Addition: $\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$

Subtraction: $\frac{a}{d} - \frac{b}{d} = \frac{a-b}{d}$

NOTE: to add or subtract fractions, you need a COMMON DEMOMINATOR!

Multiplication: $\frac{a}{c} \times \frac{b}{d} = \frac{ab}{cd}$

Division: $\frac{a}{c} \div \frac{b}{d} = \frac{ad}{bc}$

NOTE: to divide fractions, just Cross Multiply $\frac{a}{c} \times \frac{b}{d}$

Fractions to Decimals: $\frac{n}{d} = n \div d$

NOTE: to convert a fraction to a decimal, just divide the numerator by the denominator!

Decimals to Fractions:

NOTE: recognize .25, $.\overline{33}$, .5, $.\overline{66}$, .75!

Reciprocals: The reciprocal of $\frac{n}{d}$ is $\frac{d}{n}$

NOTE: A number times its reciprocal is 1!

$$\frac{2}{3} \times \frac{3}{2} = \frac{(2)(3)}{(3)(2)} = \frac{6}{6} = 1$$

Place Value: 2345.67890

5 is in the ***ones*** place
4 is in the ***tens*** place
3 is in the ***hundreds*** place
2 is in the ***thousands*** place

6 is in the ***tenths*** place
7 is in the ***hundredths*** place
8 is in the ***thousandths*** place
9 is in the ***ten-thousands*** place

Scientific Notation: $431{,}000{,}000 = 4.31 \times 10^{8}$ $\quad$ $0.0000431 = 4.31 \times 10^{-5}$

Numbers and Quantity

Factors, Multiples and Remainders

Factors: ***Positive integers that evenly divide into a number***

Example: the Factors of 36 are 1,2,3,4,6,12,18 and 36

Multiples: ***Any number that can be divided by the original number evenly (with no remainder)***

Example: Multiples of 3 are 3,6,9,12,15,18,21,…

NOTE*: if x is a factor of y and z is a multiple of y then x is a factor of z and z is a multiple of x*
Example*: 4 is a factor of 12 and 36 is a multiple of 12, so 4 is a factor of 36 and 36 is a multiple of 4!*

Remainders: ***The result "left over" after a division***

Example: 7÷3 = 2 with a remainder of 1

NOTE*: a remainder is always smaller than the dividend*

Prime Numbers: ***A positive integer greater than 1 with exactly two factors, 1 and itself***

Example: 2,3,5,7,11,13,17,19,23,… are the first nine primes

NOTE*: 1 is NOT a prime*

Problem 1: Which of the following can not be a reminder when dividing by 5? 0,1,3,4 or 5

Problem 2: What is the least common multiple of 2, 3 and 4?

Problem 3: What is a prime number between 70 and 80?

Numbers and Quantity

Ratios, Proportions and Percents

Ratios: ***The relationship between two quantities***

Example: My favorite book has 400 words per page, therefore, the ratio of words to pages is 400 to 1

NOTE:
Ratios can be expressed as a fraction with a colon **400:1**
or verbally **400 to 1**

Proportions: ***An equation in which the two expressions are ratios***

Example: $\frac{x}{40} = \frac{12}{25}$

NOTE: *Often proportions are given in which an unknown must be found (Cross multiply and solve)*

Percents: ***Percent is a ratio in which the second quantity is 100***

Example: 35% means the ratio of 35 to 100 or 35:100 or $\frac{35}{100}$

NOTE:
Percent means "Parts of" "100"

Problem 1: Solve the following for x $\frac{x}{40} = \frac{12}{50}$

Problem 2: If a sale is buy two and get one free, what is the percent savings?

Problem 3: The length of a rectangle is three times the width. If the perimeter is 30 ft, what is the width?

Numbers and Quantity

Sequences

Sequences: ***A sequence is an ordered list of numbers following a pattern***

Example: 4,7,10,13,16,… is a sequence in which each value is 3 more than the previous value. In this example, the nth term is equal to $4 + 3(n-1)$.

Example: 4,16,64,256,… is a sequence in which each value is 4 times the previous value. In this example, the nth term is equal to 4^n. This sequence is geometric sequence that has exponential growth.

Problem 1: Given a sequence of numbers {1,4,7,10,13, …}, what is the 10th term? What is the sum of the 20th and 21st terms?

Problem 2: Given a sequence of numbers {3,6,12,24,48,…}, what is the 7th term? What is the 10th term? What is the average of the 8th and 9th terms?

Numbers and Quantity

Sets

Set: ***A collection of elements*** ***NOTE: A set is denoted using brackets { }***

Example: A = {2,4,6,8,10,12,…} is the set of positive integers

Example: B = {n | n is divisible by 3} is the set of all multiples of 3

Union: ***The Union of two sets is the collection of elements in EITHER set***

Example: A = {2,3,5,7} and B = {2,4,6,8}, ***NOTE: Union is denoted* U**

***then the union of A and B is {2,3,4,5,6,7,8} and is denoted A* U B**

Intersection: ***The Intersection of two sets is the collection of elements in BOTH sets***

Example: A = {2,3,5,7} and B = {2,4,6,8}, ***NOTE: Intersection is denoted* ∩**

***then the intersection of A and B is {2} and is denoted A* ∩ B**

Problem 1: What is the intersection of all prime numbers and all even integers?

Problem 2: Given A={1,2,4,8,16,32} and B={2,3,6,12,24,48}, what is A* U B *and A* ∩ B *?

Numbers and Quantity

Counting Problems

A counting problem attempts to determine the number of possible ways elements of groups can be selected. For example, If there are two groups of children, the first group has 3 boys and the second group has 2 girls, how many different ways can one boy and one girl be selected? For each boy, there are two possible choices from the group of girls, so there are 6 possible combinations!

Permutation: ***A selection of elements in which each element can be selected only once***

Example: using the numbers {1,2,3,4} how many 3 number codes can be made? There are four possible choices for the first number, three for the second number (because the first number can not be reused) and two for the third number (because the first two numbers can not be reused), To the number of code is 4x3x2 = 24!

Combination: ***A selection of elements in which order matters (e.g. ab = ba is the same element)***

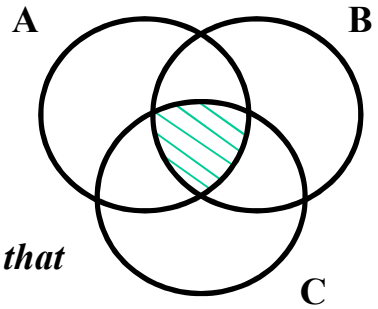
Example: Using the letters {a,b,c,d} how many 2 letter combinations can be made? The possible 2-letter permutations are ab,ac,ad,ba,bc,bd,ca,cb,cd,da,db,dc. There are 12 = 4x3 possible 2-letter permutations, but only 6 combinations!

Problem 1: Given the set {v,w,x,y,z} how many 2 letter permutations can be made? How many three letter permutations can be made?

Problem 2: Given the set {v,w,x,y,z} how many 2 letter combinations can be made? How many three letter combinations can be made?

Numbers and Quantity

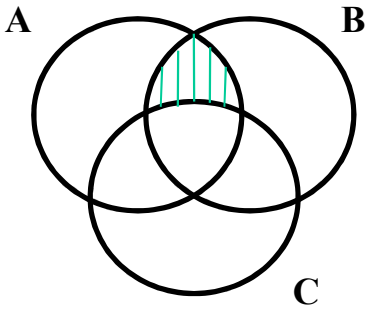
Logical Reasoning



Venn Diagram: *A graphical representation of sets as intersection circles*

Example: Let A be the set of all even numbers, B be the set of perfect squares and C be powers of 2. Name two elements that could be in the shaded region.

Set A = {0,2,4,6,8,10,12,…}
Set B = {0,1,4,9,16,25,36,49,64,81,100, …}
Set C = {1,2,4,8,16,32,64,128,…}
The shaded region is the intersection of all three sets, so elements in the shaded region, must be in all three sets. Therefore possible answers are {4,16,64,256,1024,…}.



Problem 1: Using the diagram to the right, let A be the set of all odd numbers, B be the set of multiples of 6, and C be perfect squares. Of the following numbers, which are in the shaded region? 1, 9, 16, 36, 81, 216, 256, 512