Praxis Core Academic Skills for Educators

# Math Review

# Geometry

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# Geometry and Measurement

## TOPICS

### *Fundamentals*

- *Points*
- *Lines and Line Segments*
- *Planes*
- *Angles*
- *Congruence*

### *Shapes*

- *Triangles*
- *Right Triangles*
- *Quadrilaterals*
- *Squares*
- *Rectangles*
- *Trapezoid*
- *Polygons*
- *Circles*
- *Congruent Shapes*
- *Similar Shapes*
- *Combining Shapes*

### *Angles*

- *Right, Acute, Obtuse and Straight*
- *Supplementary*
- *Complementary*
- *Vertical*

### *Formulas*

- *Perimeter*
- *Circumference*
- *Area*
- *Volume*
- *Surface Area*

### *X-Y Coordinate Plane*

- *Ordered Pairs*
- *Graphing Linear Equations*
- *Distance and Midpoint Formulas*
- *Transforms*

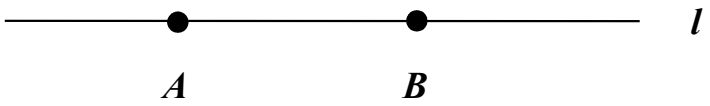
# Geometry and Measurement

## *Geometric Notation*

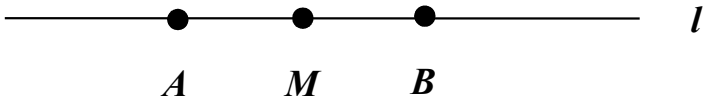
| | |
|---|---|
| $\overleftrightarrow{BF}$ | *The line containing point B and F* |
| $\overline{BF}$ | *The line segment with endpoints B and F* |
| $BF$ | *The length of line segment BF* |
| $\overrightarrow{BF}$ | *The ray starting at B and extending infinitely through F* |
| $\angle ABF$ | *The angle formed by* $\overline{AB}$ *and* $\overline{BF}$ |
| $m\angle ABF$ | *The measure of angle ABF* |
| $\triangle ABF$ | *The triangle with vertices A, B and F* |
| $ABFG$ | *The quadrilateral with vertices A, B, F and G* |
| $\overline{AB} \perp \overline{FG}$ | *AB is perpendicular to FG* |

# Geometry and Measurement

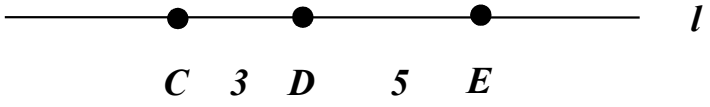
## *Points and Lines*




*unique line l, containing points A and B*




*M is the midpoint of $\overline{AB}$, so AM = MB*




*CD = 3 and DE = 5, so CE = 3+5 = 8*

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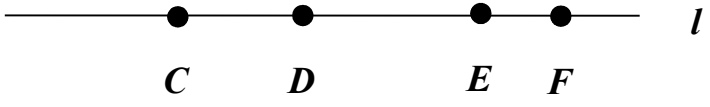
***Problem 1*: A,B and C all lie on the same line *l*, if C is the midpoint of $\overline{AB}$ and AB = 12, what is AC?**

***Problem 2*: On the line *l* above, if CD = 4, EF = 2 and CF = 10, what is the value of DE?**

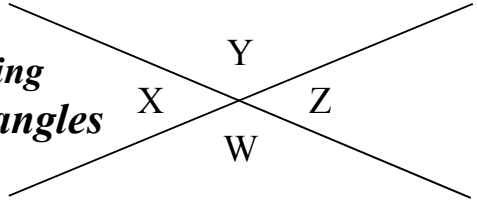


# Geometry and Measurement

## *Angles in the Plane*

***Opposite angles formed by intersecting lines are equal and are called vertical angles***

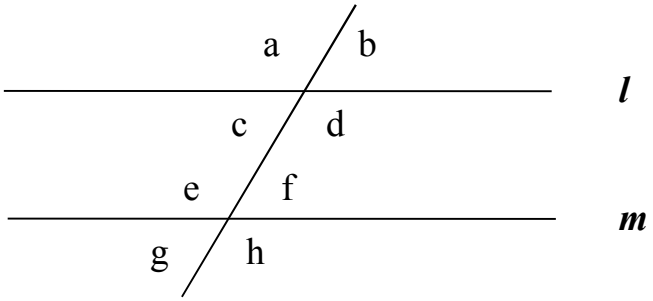
*So, X = Z and W = Y*




***Supplementary angles are Straight Angles and are equal to 180 degrees***

*So, X +W = 180º,*
*X+Y = 180º,*
*W+Z = 180º,*
*Y+Z = 180º*

***Problem 1: In the above diagram, if X is equal to 40 degrees, what is the value of W? What is the value of Z? What is the value of Y?***



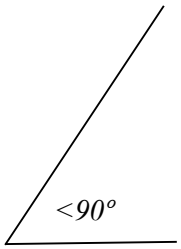

***If two parallel lines (l and m) are intersected by a third line, the alternate interior angles are equal for example, e and d are alternate interior angles***

***Problem 2: In the above diagram, l and m are parallel, name all angles that are equal to angle d? Name all angles that are supplementary to angle b?***

# Geometry and Measurement

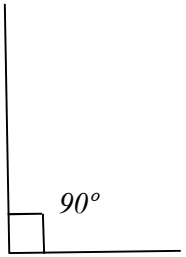
## *More on Angles in the Plane*

***An Acute Angle is less than 90°***



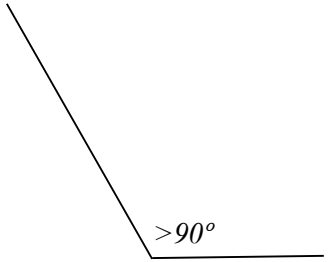

***Acute Angle***

***A Right Angle is 90°***




***Right Angle***

***An Obtuse Angle is greater than 90°***



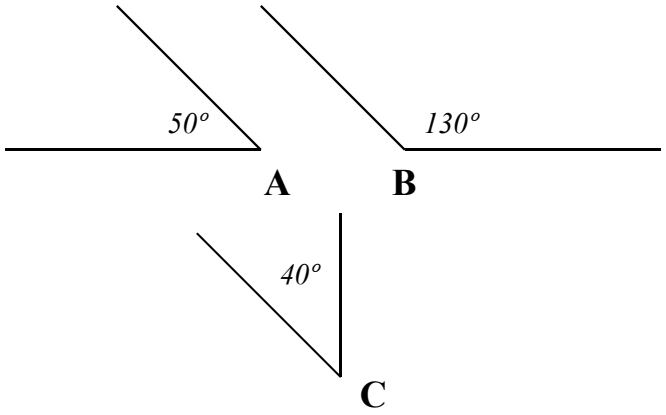

***Obtuse Angle***

***Supplementary Angles add up to 180°***

Angles A and B are **Supplementary**
Since 50 + 130 = **180**

***Complementary Angles add up to 90°***

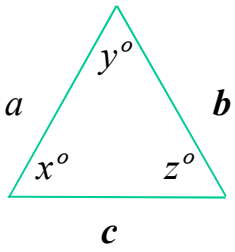
Angles A and C are **Complementary**
Since 50 + 40 = **90**



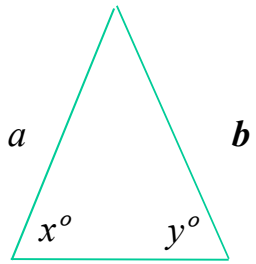

***Problem 1: If measure of angle E is 35° and angles E and D are complementary, what is the measure of angle D?***

# Geometry and Measurement

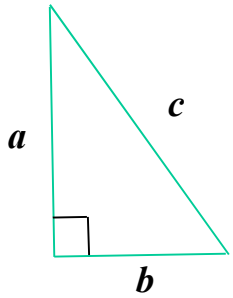
## *Triangles*




***Equilateral Triangle***
***equal sides (a=b=c)***
***equal angles (x=y=z=60)***
***Angles measure 60º***




***Isosceles Triangle***
***two equal sides (a=b)***
***two equal angles (x=y)***




***Right Triangle***
***one angle is 90º (a = 90)***
***two sides are perpendicular***
***$a^2 + b^2 = c^2$ (Pythagorean Theorem)***

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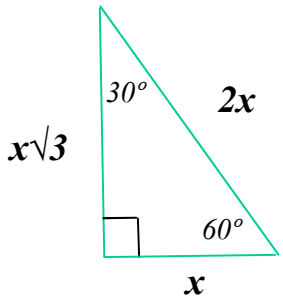
***Problem 1: If ABC is an Isosceles Triangle, such that $\angle ABC = \angle BAC$ and $m\angle ABC$ is 40º, what is the $m\angle ACB$?***

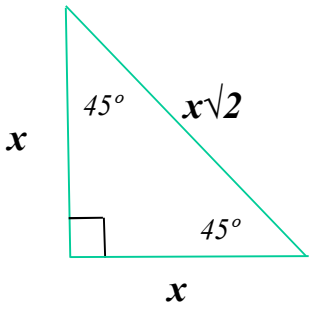
***Problem 2: If ABC is a Right Triangle, such that $m\angle ABC$ is 35º, what is the $m\angle ACB$ if it is not 90º?***

# Geometry and Measurement

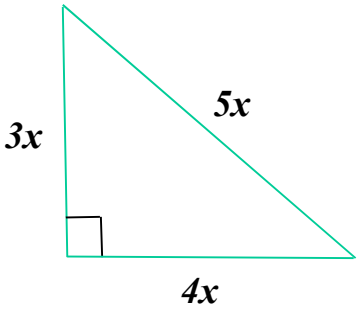
## *Triangles*




***30º-60º-90º Triangle***




***45º-45º-90º Triangle***




***3-4-5 Triangle***

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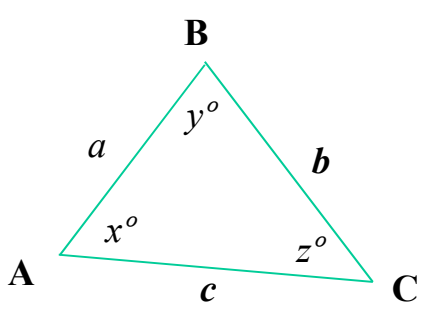
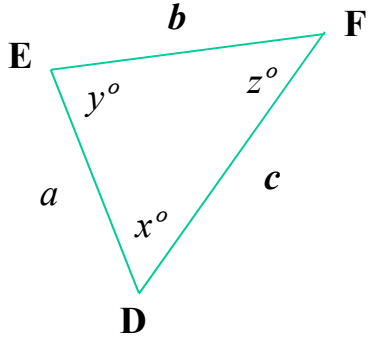
***Problem 1: If ABC is a Right Triangle, such that m∠ABC is 45º and AC = 4, what is the length of the longest side?***

# Geometry and Measurement

## *Congruent Triangles*

***triangles that have the same size and shape***



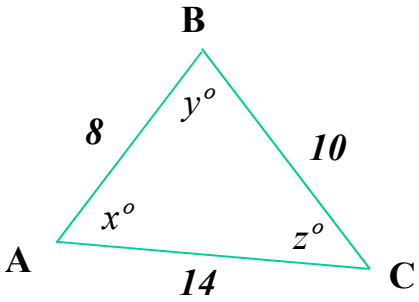

$\triangle ABC = \triangle DEF$

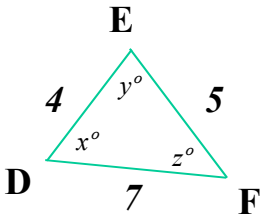
$AB = DE = a$
$BC = EF = b$
$AC = DF = c$

***Problem 1: If ABC and DEF are congruent triangles, and AB=5 and BC=15, what is EF?***

## *Similar Triangles*

***triangles that have the same shape***
***(corresponding angles are equal)***




$\triangle ABC$ *and* $\triangle DEF$
***are similar triangles***

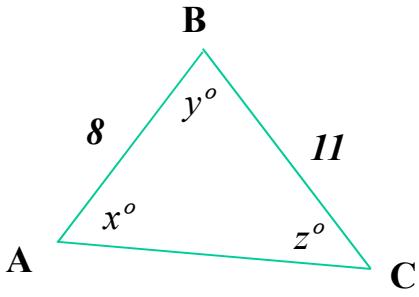
***sides are proportional***

***Problem 2: If ABC and DEF are similar triangles, and AB=5, BC=7 and DE=15, what is EF?***

# Geometry and Measurement

## *Triangle Inequality*

***The sum of the lengths of any two sides of a triangle is greater than the length of the third side***



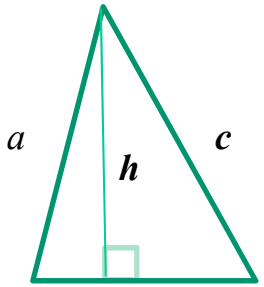

$AC < 8 + 11$

***Problem 1: In ABC, AB = 3 and AC = 7, can BC be 4? Can BC be 12? What are the ranges of values of BC?***

## *Triangle Perimeter and Area*

***Perimeter = b + a + c***
***(sum of the three sides)***

$Area = \frac{1}{2}bh$

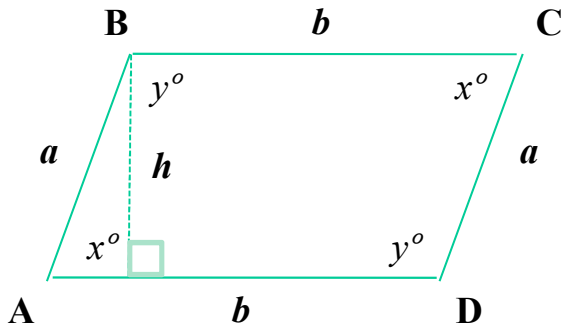



***b = base***

***Problem 2: In the above triangle, if a=6, b= 4, c=7 and h=5, what is the perimeter? What is the area?***

# Geometry and Measurement

## Quadrilaterals



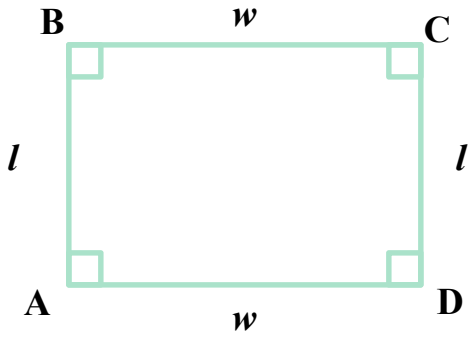

***Parallelogram***

*opposite angles are equal*

*opposite sides are equal*

$P = 2a + 2b$

$A = bh$



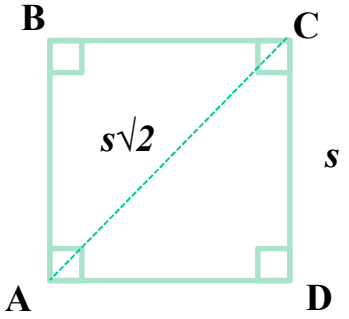

***Rectangle***

*A parallelogram with right angles*

$P = 2w + 2l$

$A = lw$

$AC = BD = \sqrt{(l^2+w^2)}$



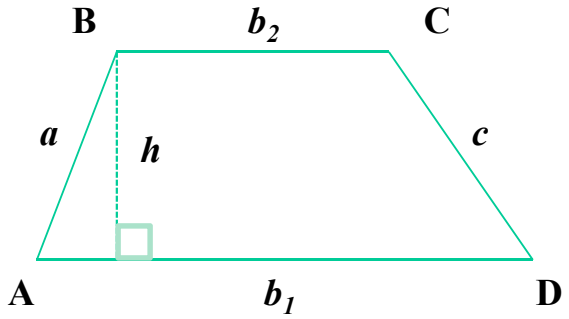

***Square***

*A rectangle with four equal sides*

$P = 4s$

$A = s^2$

$AC = BD = s\sqrt{2}$




***Trapezoid***

*only one set of*

*opposite sides are parallel*

$P = a+c+b_1+b_2$

$A = h(b_1+b_2)/2$

## Other Polygons

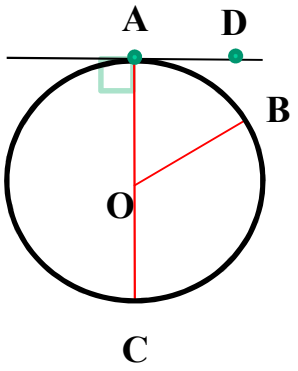
***A Regular Polygon is a polygon with all sides and angles equal***

***Determine unknown angle and sides using triangles***

| *sides* | *interior angle sum* |
|---|---|
| *3* | *180°* |
| *4* | *360°* |
| *5* | *540°* |
| *:* | *:* |
| *n* | *180(n-2)°* |

# Geometry and Measurement

## *Circles*




**O** = *Origin of Circle – the center*

**OA** = **OB** = *Radius of Circle*

**AB** = *Arc*

**AC** = *Diameter of Circle (twice the radius OA or OB)*

*the line segment* **AD** *is tangent to the circle at point* **A**.
**AD** *touches the circle at only point* **A**.

*The* ***Diameter*** *of a circle is twice the* ***Radius*** *of the circle*

$d = 2r$

*The* ***Circumference*** *of a circle is the distance around the circle – it is analogous to perimeter of a polygon*

$C = \pi d = 2 \pi r$

*The* ***Area*** *of a circle is the amount of space within the circle –*

$A = \pi r^2$

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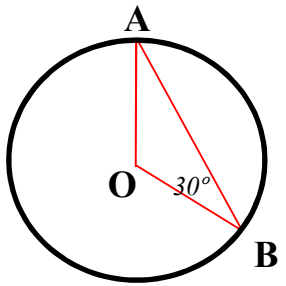
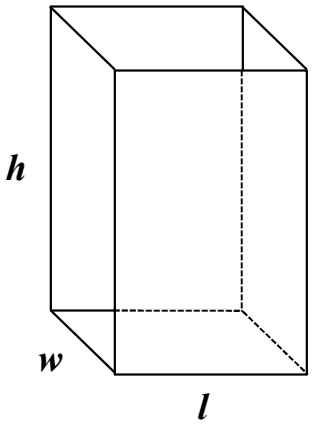
***Problem 1: Given a circle with center O and area 16π. Points A and B are on the circle and angle OBA is 30°. Find the length of line segment AB.***

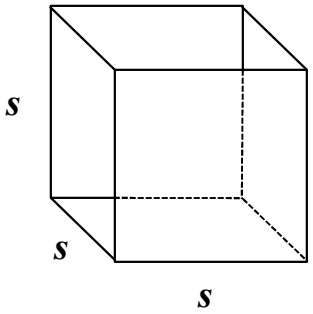


# Geometry and Measurement

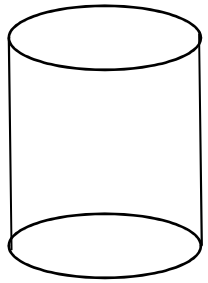
## *Solid Figures*



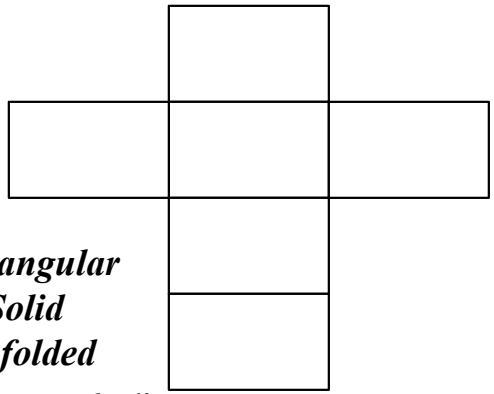

***Rectangular Solid***
***Think of a Cardboard Box***
$V = lwh$
$SA = 2lw + 2lh + 2wh$



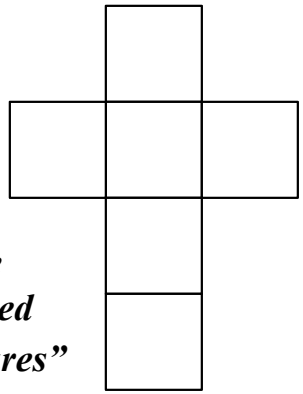

***Cube***
***A Special Rectangular Solid in which l=w=h=s***
$V = s^3$
$SA = 6s^2$




***Cylinder***
***Think of a can of soup***
$V = \prod r^2 h$
$SA = 2\prod r^2 + 2\prod rh$

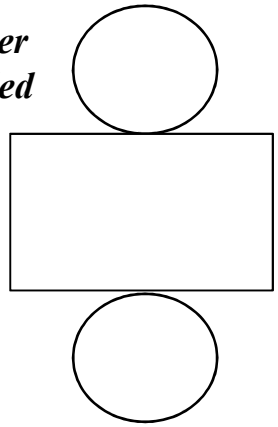


***Rectangular Solid Unfolded***
***"six rectangles"***



***Cube Unfolded***
***"six squares"***

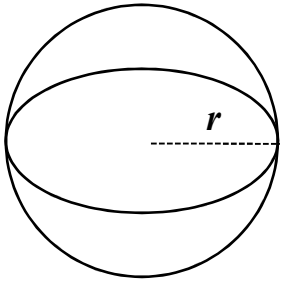
***Cylinder Unfolded***



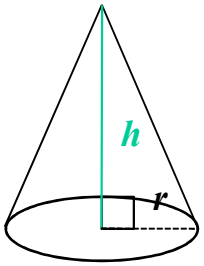
***"circle on top circle on bottom, rectangle in the middle"***

# Geometry and Measurement

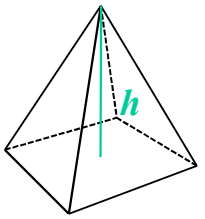
## *Solid Figures*




***Sphere***
***Think of a ball***
***All radii are equal***




***Cone***
***$V = (1/3)\prod r^2h$***
***Its Volume is 1/3 of a cylinder with the same height and base***

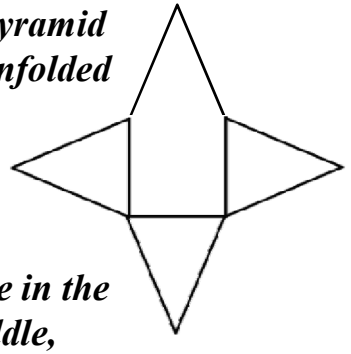



***Pyramid***
***A square at the base with four triangles***
***$V = s^2h/3$***

***Problem 1: If the volume of a cube is 125 $in^3$, what is the length of a side? What the Surface Area of the cube?***

***Problem 2: If two cylinders have equal volume and the taller is four times higher than the shorter, what is the ratio of the radii?***

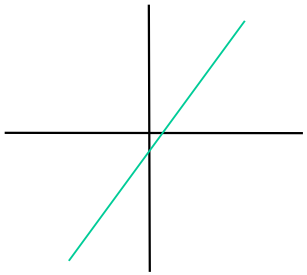
***Pyramid Unfolded***



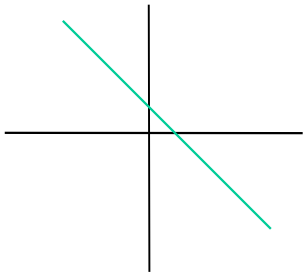
***"square in the middle, with four congruent triangles"***

# Geometry and Measurement

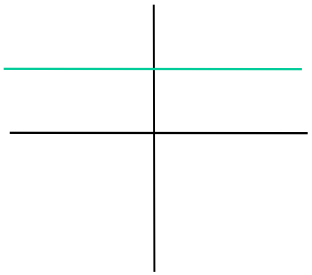
## *Coordinate Geometry*



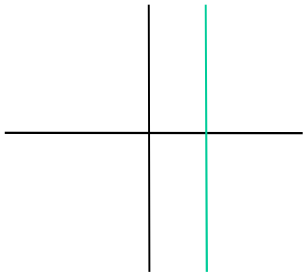
***Positive Slope***
***y = 2x - 1***



***Negative Slope***
***y = -2x + 2***



***Zero Slope***
***y = 5***



***Undefined Slope***
***x = 4***

*Two lines are **parallel** when their slopes are the same*
*$y = 2x + 3$ is parallel to $y = 2x - 7$ since the slope of both lines is 2*

*Two lines are **perpendicular** when their slopes are negative reciprocals OR the product of the slopes is -1*
*$y = -2x + 3$ is perpendicular to $y = (1/2)x - 7$*
*since $(-2)(1/2) = -1$ OR $(-2)$ is the negative reciprocal of $(1/2)$*

***Problem 1: Give a line that is parallel to the line y = 3x – 4. Give a line that is perpendicular to it.***

**Midpoint Formula**
*Given two points $(x_1, y_1)$ and $(x_2, y_2)$*
*their midpoint is $(x_m, y_m)$*
*where $x_m = (x_1+x_2)/2$ and $y_m = (y_1+y_2)/2$*

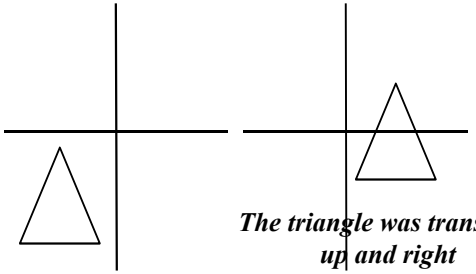
**Distance Formula**
*Given two points $(x_1, y_1)$ and $(x_2, y_2)$*
*their distance is $d$*
*where $d = \sqrt{[(x_1-x_2)^2+(y_1-y_2)^2]}$*

***Problem 2: What is the distance of the two points (1,4) and (-1,-2)? What is their midpoint?***

***Problem 3: If (3,2) is the midpoint of two points, one being (-1,-2), what is the other point?***

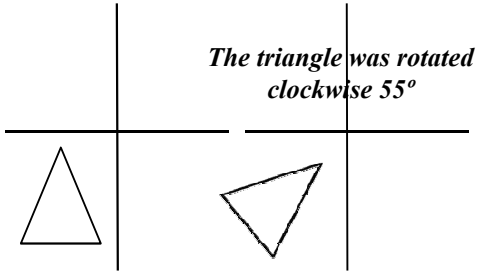
# Geometry and Measurement

## *Transformations*



*The triangle was translated up and right*

***Translation***
***Moves up/down and left/right***

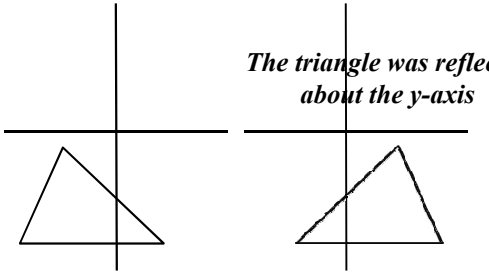


*The triangle was rotated clockwise 55°*

***Rotation***
***Rotates on a point Not necessarily the center***

*counter-clockwise*

*clockwise*



*The triangle was reflected about the y-axis*

***Reflection***
***Reflects along a line of symmetry***

***Problem 1: If a clock is rotated 90 degrees clockwise, what number will be at the top?***

***Problem 2: If the triangle to the right is reflected about the y-axis, what are the new co-ordinates? If reflected about the x-axis, what are the new co-ordinates?***

***Problem 3: If the triangle to the right is translated 2 units up and 3 units left, what are the new co-ordinates? If then (after translation) it is reflected about the x-axis, what are the new co-ordinates?***