

Praxis Core Academic Skills for Educators

# Math Review

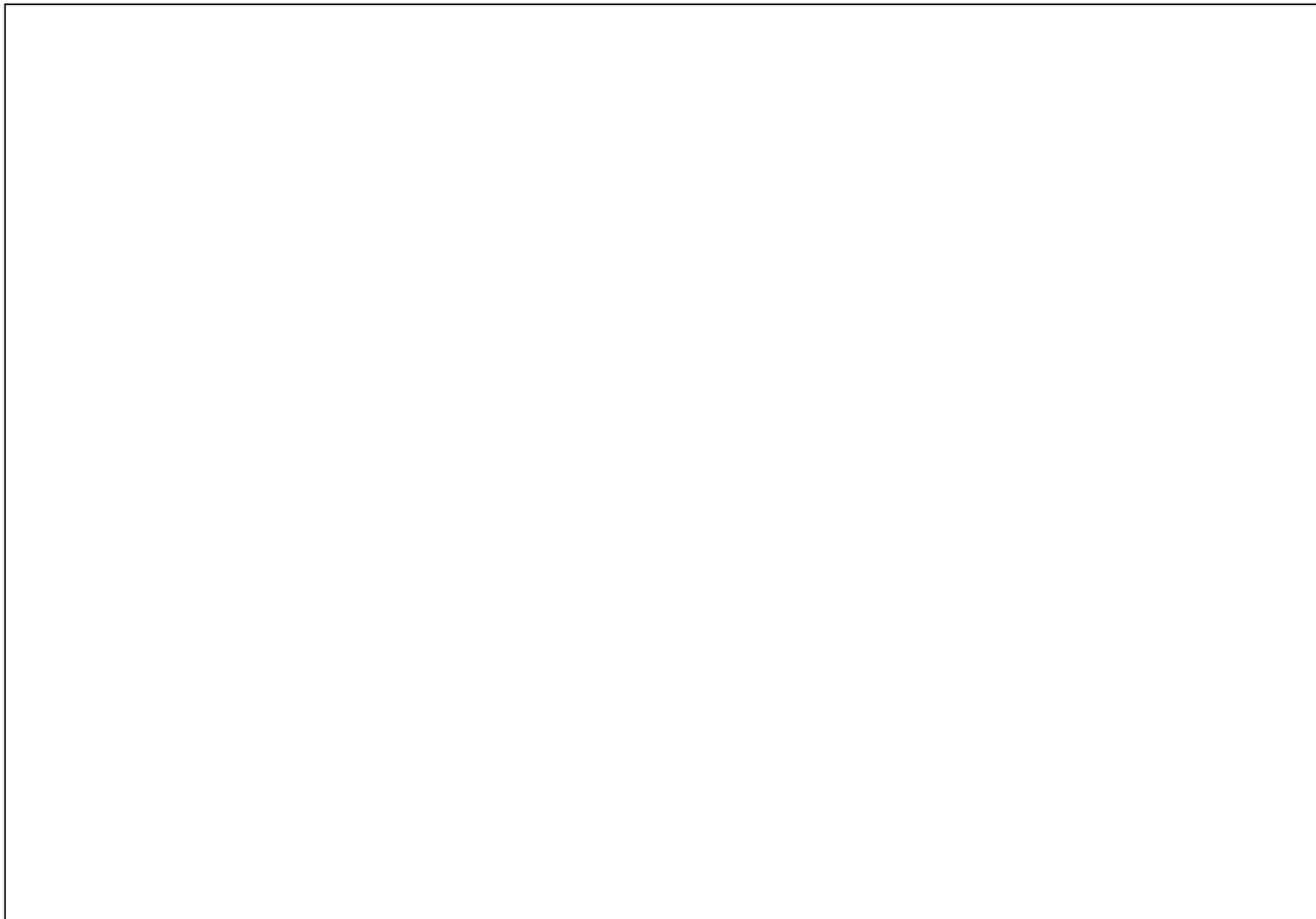
## Algebra and Functions

John L. Lehet

[jlehet@mathmaverick.com](mailto:jlehet@mathmaverick.com)

[www.mathmaverick.com](http://www.mathmaverick.com)

# Algebra and Functions



# Algebra and Functions

*Operations on Algebraic Expressions*

*Factoring*

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# Algebra and Functions

## *Algebraic Expressions*

### Identify, Group and Combine Like Terms

$$2x + 5x = 9x$$

$$10a + 3b - 6a - (-2b) + 5a = 9a + 5b$$

NOTE:

Remember Order of Operations – **PEMDAS**

**P**arenthesis

**E**xponents

**M**ultiplication/**D**ivision (left to right)

**A**ddition/**S**ubtraction (left to right)

### Multiply two Binomials

$$(x + 2)(x + 7) = x^2 + 9x + 14$$

$$(x - 3)(x + 4) = x^2 + x - 12$$

**FOIL**

### Cancel and Factor Like Terms

$$\frac{12xy^2}{4xy} = 3y$$

**Problem 1:** Simplify the expression  $5a + 3b - 6a^2 - 4b + 2a - (-2b)$

**Problem 2:** Simplify the expression  $\frac{18x^3y^2}{4xy^3}$

# Algebra and Functions

## *Factoring*

### **Difference of Two Squares**

$$a^2 - b^2 = (a + b)(a - b)$$

example

$$x^2 - 25 = (x + 5)(x - 5)$$

$$4a^2 - 9b^2 = (2a + 3b)(2a - 3b)$$

### **Common Factors**

$$3x^2 + 6x = 3x(x + 2)$$

each term has a “3x” in common  
that can be “taken out”

### **Factoring Quadratics**

$$x^2 + 4x + 3 = (x + 3)(x + 1)$$

try to factor a quadratic  
into the product of two binomials

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**Problem 1:** Completely factor the expression  $5x^3 + 10x^2 - 15x$

**Problem 2:** Completely factor the expression  $4a^3 - 4ab^2$

# Algebra and Functions

## *Exponents*

$$x^3 = x \cdot x \cdot x$$

$$x^{-2} = \frac{1}{x^2} = \frac{1}{x} \cdot \frac{1}{x}$$

$$x^{a/b} = \sqrt[b]{x^a}$$

$$x^{1/2} = \sqrt{x}$$

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

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**Problem 1:** Simplify the expression  $\frac{(a^3 \cdot a^2)^4}{a^7}$

**Problem 2:** Simplify the expression  $(a^{-5} \cdot a^2)^3$

**Problem 3:** If  $y = x^{-2/3}$ , what is the value of  $y$  if  $x = 8$ ?

# Algebra and Functions

## *Solving Equations*

**Sometimes equations may need to be factored or simplified in order to make them “look solvable”**

if  $x + 2y = 5$ , what is the value of  $3x + 6y$ ?

1. simplify  $3x + 6y$  to  $3(x + 2y)$
2. substitute 5 in for “ $x + 2y$ ” to get  $3(5) = 15$

### **Solving for one variable in terms of another**

if  $x + y = z$ , what is  $x$  in terms of  $y$  and  $z$

1. solve for  $x$  by isolating it to one side
2. subtract  $y$  from both sides to get  $x = z - y$

### **Solving equations involving radical expressions**

$$4\sqrt{a} + 12 = 24$$

1. isolate the radical by subtracting 12 and dividing by 4 ( $\sqrt{a} = 3$ )
2. square both sides ( $a = 9$ )

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**Problem 1:** If  $2x + 3y = 4$ , what is the value of  $12x + 18y$ ?

**Problem 2:** If  $2x + 3y = z$ , what is  $y$  in terms of  $x$  and  $z$ ?

**Problem 3:** Solve  $3\sqrt{a} - 7 = 8$

# Algebra and Functions

## *Absolute Value*

NOTE: The Absolute Value of a number is always greater than or equal to 0!

1. If  $|x| = 7$ , then  $x = 7$  or  $x = -7$
2. If  $|x+1| = 3$ , then  $x+1 = 3$  or  $x+1 = -3$
3. If  $|x-1| = 12$ , then  $x-1 = 12$  or  $x-1 = -12$

**In each example, there are two cases, both need to be solved**

### **Example 3:**

**Case 1:**  $x-1 = 12$   
so  $x = 13$

**Case 2:**  $x-1 = -12$   
implies  $x = -11$

Therefore, if  $|x-1| = 12$ , then  $x = 13$  or  $x = -11$

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**Problem 1:** If  $|3x - 2| = 12$ , what are the possible values of  $x$

**Problem 2:** If  $|2x + 7| = 3x - 2$ , what are the possible values of  $x$ ?



# Algebra and Functions

## *Direct Translation into Mathematical Expressions*

Look for Key Words!

NOTE: Be careful with division and subtraction, because order matters!

### ADDITION

sum  
added to  
increased by  
more than  
and

### SUBTRACTION

difference  
taken away  
decreased by  
less than

### MULTIPLICATION

times  
twice  
product  
of

### DIVISION

divided  
half  
quotient

### Verbal Expression/Equation

“The sum of two numbers is 7”  
“3 less than a number”  
“twice a number and 3”  
A number divided by twice another number

### Variable Expression/Equation

$x + y = 7$   
 $x - 3$   
 $2x + 3$   
 $x \div 2y$

NOTE: “is” means “=”

**Problem 1:** If the product of two numbers is 18 and one number is twice the other, what are the numbers?

**Problem 2:** A number is decreased by half of another number?

# Algebra and Functions

## *Inequalities*

> greater than

< less than

≥ greater than or equal

≤ less than or equal

**Simplify an inequality as if it were an equation**

$$3x + 2 > 17$$

$$3x > 15$$

$$x > 5$$

Subtract 2 from both sides

Divide both sides by 3

**NOTE: When dividing by a negative number, the inequality sign “flips”!**

$$-3x + 2 > 17$$

$$-3x > 15$$

$$x < -5$$

Subtract 2 from both sides

Divide both sides by 3 and Flip the inequality

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**Problem 1:** Which of the following are values for x if  $3x - 1 \geq 20$ ? 6, 7, 8, 9

**Problem 2:** Which of the following are values for x if  $-5x + 2 \geq -x + 10$ ? -6, -5, -4, -3, -2

**Problem 3:** Solve the following for x:  $-3 > 2x + 5 \geq 11$

# Algebra and Functions

## *Systems of Linear Equations*

### Two Linear Equations with Two Variables

$$3x + 2y = 12$$

$$5x - 4y = -2$$

The solution is the ordered pair (x,y) that makes both equations true

### Combine to make One Equation with One Variable and Solve

$$6x + 4y = 24 \quad \text{Multiply first equation by 2 (both sides)}$$

$$5x - 4y = -2$$

$$11x + 0y = 22 \quad \text{Add the equations}$$

$$x = 2 \quad \text{Solve for the variable}$$

### Use this Solution to find the other variable via Substitution

$$3x + 2y = 12$$

$$6 + 2y = 12$$

$$2y = 6$$

$$y = 3$$

So the answer is (2,3)

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**Problem 1:** Given that  $x + y = 7$  and  $2x - y = 5$ , what are the values of x and y?

**Problem 2:** The sum of two weights is 100 pounds and their difference is 20, what is the smaller weight?

# Algebra and Functions

## *Solving Quadratic Equations*

Quadratics will typically factor into the product of two binomials

$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

Factoring out of like terms may be required

$$4x^2 + 12x - 40 = 4(x^2 + 3x - 10) = 4(x + 5)(x - 2)$$

$$3x^3 + 9x^2 - 30x = 3x(x^2 + 3x - 10) = 3x(x + 5)(x - 2)$$

To solve a quadratic equation, set the quadratic to 0

$$x^2 + 3x - 6 = 4$$

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

So

$$x + 5 = 0 \quad \text{and} \quad x - 2 = 0$$

Solve Both Equations

$$x = -5 \quad \text{and} \quad x = 2$$

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**Problem 1:** Factor  $x^2 - 2x - 15$

**Problem 2:** Factor  $2x^3 + 12x^2 + 16x$

**Problem 2:** What are the solutions of  $x$  for the equation  $x^2 + 5x - 14 = 0$

# Algebra and Functions

## *Rational Equations and Inequalities*

A rational expression is the quotient of two polynomials

$$\frac{2x + 8}{3x - 2}$$

A rational equation is an equation with at least one rational expression

$$3 = \frac{2x + 8}{3x - 2}$$

Solve a rational equation by multiplying by the denominator

$$\begin{aligned}3(3x - 2) &= 2x + 8 \\9x - 6 &= 2x + 8 \\7x &= 14 \\x &= 2\end{aligned}$$

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**Problem 1:** Solve  $\frac{x + 3}{2x - 3} = 2$

**Problem 2:** Solve  $\frac{2x - 9}{3x - 5} = -5$

# Algebra and Functions

## *Direct and Inverse Variation*

**Variables  $x$  and  $y$  are directly proportional if  $y = kx$  for some constant value  $k$**

If  $x$  increases then  $y$  increases

If  $x$  decreases then  $y$  decreases

if  $y$  increases then  $x$  increases

If  $y$  decreases then  $x$  decreases

**$x$  and  $y$   
act the same!**

**Variables  $x$  and  $y$  are inversely proportional if  $y = k/x$  for some constant value  $k$**

If  $x$  increases then  $y$  decreases

If  $x$  decreases then  $y$  increases

if  $y$  increases then  $x$  decreases

If  $y$  decreases then  $x$  increases

**$x$  and  $y$   
act as opposites!**

# Algebra and Functions

## *Functions*

$f(x) = x + 1$  is a function of  $x$

**Domain of a**

**function:** *The set of all  $x$  values in which the function is defined*

*Example: the domain of  $f(x) = x + 1$  is all real numbers*

*Example: the domain of  $f(x) = 1/(x - 1)$  is all real numbers except for 1*

*Example: the domain of  $f(x) = \sqrt{x}$  is all nonnegative real numbers*

**Range of a**

**function:** *The set of all values in which  $f(x)$  is defined*

*Example: the range of  $f(x) = x + 1$  is all real numbers*

*Example: the range of  $f(x) = 1/(x - 1)$  is all real numbers except for 0*

*Example: the range of  $f(x) = |x|$  is all nonnegative real numbers*

**To solve a function at a specific value, just substitute**

**If  $f(x) = 2x - 2$ , what is  $f(-3)$ ?**

$$f(-3) = 2(-3) - 2 = -6 - 2 = -8$$